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The cosmological constant as an eigenvalue of a Sturm–Liouville problem and its renormalization

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Abstract
We discuss the case of massive gravitons and their relation with the cosmological constant, considered as an eigenvalue of a Sturm–Liouville problem. A variational approach with Gaussian trial wavefunctionals is used as a method to study such a problem. We approximate the equation to one loop in a Schwarzschild background and a zeta function regularization is involved to handle divergences. The regularization is closely related to the subtraction procedure appearing in the computation of the Casimir energy in a curved background. A renormalization procedure is introduced to remove the infinities together with a renormalization group equation.

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1. Introduction

There are two interesting and fundamental questions of Einstein gravity which have not received an answer yet: one of these is the cosmological constant Λ_c and the other one is the existence of gravitons with or without mass. While the massless graviton is a natural consequence of the linearized Einstein field equations, the massive case is more delicate. At the linearized level, we are forced to introduce the Pauli–Fierz mass term [1]

$$S_{PF} = \frac{m_g^2}{8\kappa}\int \mathrm{d}^4x\sqrt{-g^{(4)}}[h^{\mu\nu}h_{\mu\nu} - h^2], \tag{1}$$

where m_g is the graviton mass and $\kappa = 8\pi G$. G is the Newton constant. The Pauli–Fierz mass term breaks the symmetry $h_{\mu\nu} \longrightarrow h_{\mu\nu} + 2\nabla_{(\mu}\xi_{\nu)}$, but does not introduce ghosts. Boulware and Deser tried to include a mass in the general framework and not simply in the linearized theory. They discovered that the theory is unstable and produces ghosts [2]. Another problem

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appearing when one considers a massive graviton in Minkowski space is the limit $m_g \to 0$: the analytic expression of the graviton propagator in the massive and in the massless limit does not coincide. This is known as van Dam–Veltman–Zakharov (vDVZ) discontinuity [3]. Other than the appearance of a discontinuity in the massless limit, they showed that a comparison with experiment led the graviton to be rigorously massless. Actually, we know that there exist bounds on the graviton rest mass that put the upper limit on a value less than 10^{-62}–10^{-66} g [4]. Recently, there has been a considerable interest in massive gravity theories, especially about the vDVZ discontinuity examined in de Sitter and anti-de Sitter space. Indeed in a series of papers, it has been shown that the vDVZ discontinuity disappears in the massless, at least at the tree level approximation [5], while it reappears at one loop [6]. If we fix our attention on the positive cosmological term expanded to one loop, we can see that its structure is

$$S_{\Lambda_c} = \frac{\Lambda_c}{4\kappa}\int d^4x\sqrt{-g^{(4)}}\left[h^{\mu\nu}h_{\mu\nu} - \frac{1}{2}h^2\right], \tag{2}$$

which is not of the Pauli–Fierz form[1]. Nevertheless, we have to note that the nontrace terms of S_{PF} and S_{Λ_c} can be equal if

$$\frac{m_g^2}{2} = \Lambda_c. \tag{3}$$

In other words the graviton mass and the cosmological constant seem to be two aspects of the same problem. Furthermore, the cosmological constant suffers the same problem of smallness, because the more recent estimates on Λ_c give an order of 10^{-47} GeV^4, while a crude estimate of the zero point energy (ZPE) of some field of mass m with a cut-off at the Planck scale gives $E_{\mathrm{ZPE}} \approx 10^{71}$ GeV^4 with a difference of about 118 orders [8]. One interesting way to relate the cosmological constant to the ZPE is given by the Einstein field equations with our matter fields

$$R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R^{(4)} + \Lambda_c g_{\mu\nu} = G_{\mu\nu} + \Lambda_c g_{\mu\nu} = 0, \tag{4}$$

where $G_{\mu\nu}$ is the Einstein tensor. If we introduce a time-like unit vector u^μ such that $u\cdot u = -1$, then

$$G_{\mu\nu}u^\mu u^\mu = \Lambda_c. \tag{5}$$

This is simply the Hamiltonian constraint written in terms of the equation of motion. However, we would like to compute not Λ_c, but its expectation value $\langle\Lambda_c\rangle$ on some trial wavefunctional. On the other hand,

$$\frac{\sqrt{g}}{2\kappa}G_{\mu\nu}u^\mu u^\mu = \frac{\sqrt{g}}{2\kappa}R + \frac{2\kappa}{\sqrt{g}}\left(\frac{\pi^2}{2} - \pi^{\mu\nu}\pi_{\mu\nu}\right) = -\mathcal{H}, \tag{6}$$

where R is the scalar curvature in three dimensions. Therefore,

$$\frac{\langle\Lambda_c\rangle}{\kappa} = -\frac{1}{V}\left\langle\int_\Sigma d^3x\mathcal{H}\right\rangle = -\frac{1}{V}\left\langle\int_\Sigma d^3x\hat{\Lambda}_\Sigma\right\rangle, \tag{7}$$

where the last expression stands for

$$\frac{1}{V}\frac{\int\mathcal{D}[g_{ij}]\Psi^*[g_{ij}]\int_\Sigma d^3x\mathcal{H}\Psi[g_{ij}]}{\int\mathcal{D}[g_{ij}]\Psi^*[g_{ij}]\Psi[g_{ij}]} = \frac{1}{V}\frac{\langle\Psi|\int_\Sigma d^3x\hat{\Lambda}_\Sigma|\Psi\rangle}{\langle\Psi|\Psi\rangle} = -\frac{\Lambda}{\kappa}, \tag{8}$$

and where we have integrated over the hypersurface Σ, divided by its volume and functionally integrated over quantum fluctuation. Note that equation (8) can be derived starting with the Wheeler–De Witt equation (WDW) [9] which represents invariance under *time*

[1] For this purpose, see also [7].

reparametrization. Equation (8) represents the Sturm–Liouville problem associated with the cosmological constant. The related boundary conditions are dictated by the choice of the trial wavefunctionals which, in our case, are of the Gaussian type. Different types of wavefunctionals correspond to different boundary conditions. Extracting the TT tensor contribution from equation (8) approximated to second order in perturbation of the spatial part of the metric into a background term, $\bar{g}_{ij}$, and a perturbation, h_{ij}, we get $\hat{\Lambda}_{\Sigma}^{\perp} =$

$$\frac{1}{4V}\int_{\Sigma}\mathrm{d}^3x\sqrt{\bar{g}}G^{ijkl}\left[(2\kappa)K^{-1\perp}(x,x)_{ijkl}+\frac{1}{(2\kappa)}(\triangle_2)^a_j K^{\perp}(x,x)_{iakl}\right]. \tag{9}$$

Here G^{ijkl} represents the inverse De Witt metric and all indices run from one to three. The propagator $K^{\perp}(x,x)_{iakl}$ can be represented as

$$K^{\perp}(\overrightarrow{x},\overrightarrow{y})_{iakl} := \sum_{\tau}\frac{h_{ia}^{(\tau)\perp}(\overrightarrow{x})h_{kl}^{(\tau)\perp}(\overrightarrow{y})}{2\lambda(\tau)}, \tag{10}$$

where $h_{ia}^{(\tau)\perp}(\overrightarrow{x})$ are the eigenfunctions of $\triangle_2$, whose explicit expression for the massive case will be shown in the following section. τ denotes a complete set of indices and $\lambda(\tau)$ are a set of variational parameters to be determined by the minimization of equation (9). The expectation value of $\hat{\Lambda}_{\Sigma}^{\perp}$ is easily obtained by inserting the form of the propagator into equation (9) and minimizing with respect to the variational function $\lambda(\tau)$. Thus the total one-loop energy density for TT tensors becomes

$$\frac{\Lambda}{8\pi G} = -\frac{1}{4V}\sum_{\tau}\left[\sqrt{\omega_1^2(\tau)}+\sqrt{\omega_2^2(\tau)}\right]. \tag{11}$$

The above expression makes sense only for $\omega_i^2(\tau) > 0$, where ω_i are the eigenvalues of $\triangle_2$.

2. The massive graviton transverse traceless (TT) spin-2 operator for the Schwarzschild metric and the WKB approximation

The next step is the evaluation of equation (11), when the graviton has a rest mass. Following Rubakov [13], the Pauli–Fierz term can be rewritten in such a way to explicitly violate Lorentz symmetry, but to preserve the three-dimensional Euclidean symmetry. In Minkowski space it takes the form

$$S_m = -\frac{1}{8\kappa}\int_{\mathcal{M}}\mathrm{d}^4x\sqrt{-g}\mathcal{L}_m, \tag{12}$$

where

$$\mathcal{L}_m = m_0^2h^{00}h_{00}+2m_1^2h^{0i}h_{0i}-m_2^2h^{ij}h_{ij}+m_3^2h^{ii}h_{jj}-2m_4^2h^{00}h_{ii} \tag{13}$$

A comparison between S_m and the Pauli–Fierz term shows that they can be set equal if we make the following choice[2]

$$m_0^2 = 0 \qquad m_1^2 = m_2^2 = m_3^2 = m_4^2 = m^2 > 0. \tag{14}$$

If we fix the attention on the very special case $m_0^2 = m_1^2 = m_3^2 = m_4^2 = 0$; $m_2^2 = m_g^2 > 0$, we can see that the trace part disappears and we get

$$S_m = \frac{m_g^2}{8\kappa}\int\mathrm{d}^4x\sqrt{-\hat{g}}[h^{ij}h_{ij}] \quad\Longrightarrow\quad \mathcal{H}_m = -\frac{m_g^2}{8\kappa}\int\mathrm{d}^3xN\sqrt{\hat{g}}[h^{ij}h_{ij}]. \tag{15}$$

[2] See also Dubovski [14] for a detailed discussion about the different choices of m_1, m_2, m_3 and m_4.

Its contribution to the spin-2 operator for the Schwarzschild metric will be

$$(\triangle_2 h^{\mathrm{TT}})_i^j := -(\triangle_T h^{\mathrm{TT}})_i^j + 2(Rh^{\mathrm{TT}})_i^j + \left(m_g^2 h^{\mathrm{TT}}\right)_i^j \tag{16}$$

and

$$-(\triangle_T h^{\mathrm{TT}})_i^j = -\triangle_S (h^{\mathrm{TT}})_i^j + \frac{6}{r^2}\left(1 - \frac{2MG}{r}\right)(h^{\mathrm{TT}})_i^j. \tag{17}$$

$\triangle_S$ is the scalar curved Laplacian, whose form is

$$\triangle_S = \left(1 - \frac{2MG}{r}\right)\frac{\mathrm{d}^2}{\mathrm{d}r^2} + \left(\frac{2r - 3MG}{r^2}\right)\frac{\mathrm{d}}{\mathrm{d}r} - \frac{L^2}{r^2} \tag{18}$$

and R_j^a is the mixed Ricci tensor whose components are

$$R_i^a = \left\{-\frac{2MG}{r^3}, \frac{MG}{r^3}, \frac{MG}{r^3}\right\}. \tag{19}$$

This implies that the scalar curvature is traceless. We are therefore led to study the following eigenvalue equation:

$$(\triangle_2 h^{\mathrm{TT}})_i^j = \omega^2 h_j^i \tag{20}$$

where ω^2 is the eigenvalue of the corresponding equation. In doing so, we follow Regge and Wheeler in analysing the equation as modes of definite frequency, angular momentum and parity [10]. In particular, our choice for the three-dimensional gravitational perturbation is represented by its even-parity form

$$(h^{\mathrm{even}})_j^i(r, \vartheta, \phi) = \mathrm{diag}[H(r), K(r), L(r)]Y_{lm}(\vartheta, \phi). \tag{21}$$

Defining reduced fields and passing to the proper geodesic distance from the *throat* of the bridge, the system (20) becomes

$$\begin{cases} \left[-\dfrac{\mathrm{d}^2}{\mathrm{d}x^2} + \dfrac{l(l+1)}{r^2} + m_1^2(r)\right] f_1(x) = \omega_{1,l}^2 f_1(x) \\ \left[-\dfrac{\mathrm{d}^2}{\mathrm{d}x^2} + \dfrac{l(l+1)}{r^2} + m_2^2(r)\right] f_2(x) = \omega_{2,l}^2 f_2(x) \end{cases} \tag{22}$$

where we have defined $r \equiv r(x)$ and

$$\begin{cases} m_1^2(r) = m_g^2 + U_1(r) = m_g^2 + m_1^2(r, M) - m_2^2(r, M) \\ m_2^2(r) = m_g^2 + U_2(r) = m_g^2 + m_1^2(r, M) + m_2^2(r, M). \end{cases} \tag{23}$$

$m_1^2(r, M) \to 0$ when $r \to \infty$ or $r \to 2MG$ and $m_2^2(r, M) = 3MG/r^3$. Note that, while $m_2^2(r)$ is constant in sign, $m_1^2(r)$ is not. Indeed, for the critical value $\bar{r} = 5MG/2$, $m_1^2(\bar{r}) = m_g^2$ and in the range $(2MG, 5MG/2)$ for some values of m_g^2, $m_1^2(\bar{r})$ can be negative. It is interesting therefore to concentrate in this range, where $m_1^2(r, M)$ vanishes when compared with $m_2^2(r, M)$. So, in a first approximation we can write

$$\begin{cases} m_1^2(r) \simeq m_g^2 - m_2^2(r_0, M) = m_g^2 - m_0^2(M) \\ m_2^2(r) \simeq m_g^2 + m_2^2(r_0, M) = m_g^2 + m_0^2(M), \end{cases} \tag{24}$$

where we have defined a parameter $r_0 > 2MG$ and $m_0^2(M) = 3MG/r_0^3$. The main reason for introducing a new parameter resides in the fluctuation of the horizon that forbids any kind of approach. It is now possible to explicitly evaluate equation (11) in terms of the effective mass. To further proceed, we use the WKB method used by 't Hooft in the brick wall problem [11] and we count the number of modes with frequency less than ω_i, $i = 1, 2$. This is given approximately by ($r \equiv r(x)$)

$$\tilde{g}(\omega_i) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \mathrm{d}x \int \sqrt{k_i^2(r, l, \omega_i)}(2l + 1)\, \mathrm{d}l, \tag{25}$$

Here it is understood that the integration with respect to x and l is taken over those values which satisfy $k_i^2(r, l, \omega_i) \geqslant 0$, $i = 1, 2$. Thus the one-loop total energy for TT tensors becomes

$$\frac{\Lambda}{8\pi G} = \rho_1 + \rho_2 = -\frac{1}{16\pi^2} \sum_{i=1}^{2} \int_{\sqrt{m_i^2(r)}}^{+\infty} \omega_i^2 \sqrt{\omega_i^2 - m_i^2(r)}\, \mathrm{d}\omega_i, \tag{26}$$

where we have included an additional 4π coming from the angular integration.

3. One-loop energy regularization and renormalization

Here, we use the zeta function regularization method to compute the energy densities ρ_1 and ρ_2. Note that this procedure is completely equivalent to the subtraction procedure of the Casimir energy computation where the zero point energy (ZPE) in different backgrounds with the same asymptotic properties is involved. For this purpose, we introduce the additional mass parameter μ in order to restore the correct dimension for the regularized quantities. Such an arbitrary mass scale emerges unavoidably in any regularization scheme. Then we have

$$\rho_i(\varepsilon) = \frac{1}{16\pi^2} \mu^{2\varepsilon} \int_{\sqrt{m_i^2(r)}}^{+\infty} \mathrm{d}\omega_i \frac{\omega_i^2}{\left(\omega_i^2 - m_i^2(r)\right)^{\varepsilon - \frac{1}{2}}}. \tag{27}$$

The integration has to be meant in the range where $\omega_i^2 - m_i^2(r) \geqslant 0$. One gets

$$\rho_i(\varepsilon) = \frac{m_i^4(r)}{256\pi^2} \left[\frac{1}{\varepsilon} + \ln\left(\frac{\mu^2}{m_i^2(r)}\right) + 2\ln 2 - \frac{1}{2}\right], \tag{28}$$

$i = 1, 2$. To handle with the divergent energy density, we extract the divergent part of Λ, in the limit $\varepsilon \to 0$ and we set

$$\Lambda^{\mathrm{div}} = \frac{G}{32\pi\varepsilon} \left(m_1^4(r) + m_2^4(r)\right). \tag{29}$$

Thus, the renormalization is performed via the absorption of the divergent part into the re-definition of the bare classical constant Λ

$$\Lambda \to \Lambda_0 + \Lambda^{\mathrm{div}}. \tag{30}$$

The remaining finite value for the cosmological constant reads

$$\frac{\Lambda_0}{8\pi G} = \frac{1}{256\pi^2} \left\{ m_1^4(r) \left[\ln\left(\frac{\mu^2}{\left|m_1^2(r)\right|}\right) + 2\ln 2 - \frac{1}{2}\right] + m_2^4(r) \left[\ln\left(\frac{\mu^2}{m_2^2(r)}\right) + 2\ln 2 - \frac{1}{2}\right] \right\} = (\rho_1(\mu) + \rho_2(\mu)) = \rho_{\mathrm{eff}}^{\mathrm{TT}}(\mu, r). \tag{31}$$

The quantity in equation (31) depends on the arbitrary mass scale μ. It is appropriate to use the renormalization group equation to eliminate such a dependence. To this aim, we impose that [12]

$$\frac{1}{8\pi G} \mu \frac{\partial \Lambda_0^{\mathrm{TT}}(\mu)}{\partial \mu} = \mu \frac{\mathrm{d}}{\mathrm{d}\mu} \rho_{\mathrm{eff}}^{\mathrm{TT}}(\mu, r). \tag{32}$$

Solving it we find that the renormalized constant Λ_0 should be treated as a running one in the sense that it varies provided that the scale μ is changing

$$\Lambda_0(\mu, r) = \Lambda_0(\mu_0, r) + \frac{G}{16\pi}\left(m_1^4(r) + m_2^4(r)\right)\ln\frac{\mu}{\mu_0}. \tag{33}$$

Substituting equation (33) into equation (31), we find

$$\frac{\Lambda_0(\mu_0, r)}{8\pi G} = -\frac{1}{256\pi^2}\left\{\left(m_g^2 - m_0^2(M)\right)^2\left[\ln\left(\frac{\left|m_g^2 - m_0^2(M)\right|}{\mu_0^2}\right) - 2\ln 2 + \frac{1}{2}\right]\right.$$
$$\left. + \left(m_g^2 + m_0^2(M)\right)^2\left[\ln\left(\frac{m_g^2 + m_0^2(M)}{\mu_0^2}\right) - 2\ln 2 + \frac{1}{2}\right]\right\}. \tag{34}$$

We can now discuss three cases: (1) $m_g^2 \gg m_0^2(M)$, (2) $m_g^2 = m_0^2(M)$, (3) $m_g^2 \ll m_0^2(M)$. In case (1), we can rearrange equation (34) to obtain

$$\frac{\Lambda_0(\mu_0, r)}{8\pi G} \simeq -\frac{m_g^4}{128\pi^2}\left[\ln\left(\frac{m_g^2}{4\mu_M^2}\right) + \frac{1}{2}\right], \tag{35}$$

where we have introduced an intermediate scale defined by

$$\mu_M^2 = \mu_0^2 \exp\left(-\frac{3m_0^4(M)}{2m_g^4}\right). \tag{36}$$

With the help of equation (36), the computation of the minimum of Λ_0 is more simple. Indeed, if we define

$$x = \frac{m_g^2}{4\mu_M^2} \quad\Longrightarrow\quad \Lambda_{0,M}(\mu_0, x) = -\frac{G\mu_M^4}{\pi}x^2\left[\ln(x) + \frac{1}{2}\right]. \tag{37}$$

As a function of x, $\Lambda_{0,M}(\mu_0, x)$ vanishes for $x = 0$ and $x = \exp\left(-\frac{1}{2}\right)$ and when $x \in \left[0, \exp\left(-\frac{1}{2}\right)\right]$, $\Lambda_{0,M}(\mu_0, x) \geqslant 0$. It has a maximum for

$$\bar{x} = \frac{1}{e} \quad\Longleftrightarrow\quad m_g^2 = \frac{4\mu_M^2}{e} = \frac{4\mu_0^2}{e}\exp\left(-\frac{3m_0^4(M)}{2m_g^4}\right) \tag{38}$$

and its value is

$$\Lambda_{0,M}(\mu_0, \bar{x}) = \frac{G\mu_M^4}{2\pi e^2} = \frac{G\mu_0^4}{2\pi e^2}\exp\left(-\frac{3m_0^4(M)}{m_g^4}\right) \tag{39}$$

or

$$\Lambda_{0,M}(\mu_0, \bar{x}) = \frac{G}{32\pi}m_g^4 \exp\left(\frac{3m_0^4(M)}{m_g^4}\right). \tag{40}$$

In case (2), equation (34) becomes

$$\frac{\Lambda_0(\mu_0, r)}{8\pi G} \simeq \frac{\Lambda_0(\mu_0)}{8\pi G} = -\frac{m_g^4}{64\pi^2}\left[\ln\left(\frac{m_g^2}{2\mu_0^2}\right) + \frac{1}{2}\right] \tag{41}$$

or

$$\frac{\Lambda_0(\mu_0)}{8\pi G} = -\frac{m_0^4(M)}{64\pi^2}\left[\ln\left(\frac{m_0^2(M)}{2\mu_0^2}\right) + \frac{1}{2}\right]. \tag{42}$$

Again we define a dimensionless variable

$$x = \frac{m_g^2}{2\mu_0^2} \quad\Longrightarrow\quad \Lambda_{0,0}(\mu_0, x) = -\frac{G\mu_0^4}{2\pi}x^2\left[\ln(x) + \frac{1}{2}\right]. \tag{43}$$

The formal expression of equation (43) is very close to equation (37) and indeed the extrema are in the same position of the scale variable x, even if the meaning of the scale is different here. $\Lambda_{0,0}(\mu_0, x)$ vanishes for $x = 0$ and $x = 4\exp\left(-\frac{1}{2}\right)$. In this range, $\Lambda_{0,0}(\mu_0, x) \geqslant 0$ and it has a minimum located in

$$\bar{x} = \frac{1}{e} \quad \Longrightarrow \quad m_g^2 = \frac{2\mu_0^2}{e} \tag{44}$$

and

$$\Lambda_{0,0}(\mu_0, \bar{x}) = \frac{G\mu_0^4}{4\pi e^2} \tag{45}$$

or

$$\Lambda_{0,0}(\mu_0, \bar{x}) = \frac{G}{16\pi} m_g^4 = \frac{G}{16\pi} m_0^4(M). \tag{46}$$

Finally case (3) leads to

$$\frac{\Lambda_0(\mu_0, r)}{8\pi G} \simeq -\frac{m_0^4(M)}{128\pi^2}\left[\ln\left(\frac{m_0^2(M)}{4\mu_m^2}\right) + \frac{1}{2}\right], \tag{47}$$

where we have introduced another intermediate scale

$$\mu_m^2 = \mu_0^2 \exp\left(-\frac{3m_g^4}{2m_0^4(M)}\right). \tag{48}$$

By repeating the same procedure of previous cases, we define

$$x = \frac{m_0^2(M)}{4\mu_m^2} \quad \Longrightarrow \quad \Lambda_{0,m}(\mu_0, x) = -\frac{G\mu_m^4}{\pi} x^2 \left[\ln(x) + \frac{1}{2}\right]. \tag{49}$$

Also this case has a maximum for

$$\bar{x} = \frac{1}{e} \quad \Longrightarrow \quad m_0^2(M) = \frac{4\mu_m^2}{e} = \frac{4\mu_0^2}{e} \exp\left(-\frac{3m_g^4}{2m_0^4(M)}\right). \tag{50}$$

and

$$\Lambda_{0,m}(\mu_0, \bar{x}) = \frac{G\mu_m^4}{2\pi e^2} = \frac{G\mu_0^4}{2\pi e^2} \exp\left(-\frac{3m_g^4}{m_0^4(M)}\right) \tag{51}$$

or

$$\Lambda_{0,M}(\mu_0, \bar{x}) = \frac{G}{32\pi} m_0^4(M) \exp\left(\frac{3m_g^4}{m_0^4(M)}\right). \tag{52}$$

Remark. Note that in any case, the maximum of Λ corresponds to the minimum of the energy density.

A quite curious thing comes on the estimate on the 'square graviton mass', which in this context is closely related to the cosmological constant. Indeed, from equation (44) applied on the square mass, for the particular value of the normalization point μ_0 at the Planck scale, we get

$$m_g^2 \propto \mu_0^2 \simeq 10^{32}\,\mathrm{GeV}^2 = 10^{50}\,\mathrm{eV}^2, \tag{53}$$

while the experimental upper bound is of the order

$$\left(m_g^2\right)_{\mathrm{exp}} \propto 10^{-48} - 10^{-58}\,\mathrm{eV}^2, \tag{54}$$

which gives a difference of about 10^{98}–10^{108} orders. This discrepancy strongly recalls the difference of the cosmological constant estimated at the Planck scale with that measured in the space where we live.

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